

The power of an optical Maxwell's demon in the presence of photon-number correlations

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We study how correlations affect the performance of the simulator of a Maxwell's demon demonstrated in a recent optical experiment [Vidrighin et al., Phys. Rev. Lett. **116**, 050401 (2016)]. The power of the demon is found to be enhanced or hindered, depending on the nature of the correlation, in close analogy to the situation faced by a thermal demon.

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I. INTRODUCTION

The Maxwell's Demon, first introduced in a thought experiment by James Clerk Maxwell [1], is a being with the ability to extract work from a system in contact with a single thermal bath, in apparent violation of the second law of thermodynamics. Since everyone believes that the second law is not to be violated, a long series of exorcisms of the demon have been proposed [2, 3]. Following Landauer [4] and Bennett [5], today there is a broad consensus that information must come into the balance, and specifically that information erasure comes with an entropy cost; though some discordant voices remain (see e.g. [6] and references therein). In the last decade or so, it was noticed that the most powerful demon should be able to manipulate information at the quantum level [7–9]. Studies of information balances have provided intriguing insights [9–11], but the connection with usual thermodynamics requires a quantitative definition of work, which is a subject of controversy in the quantum regime [12–18].

One way to sort out theoretical discussions is to resort to experiments [19–23]. While such simulations of the Maxwell demon cannot be used to draw conclusions about ultimate limits, they do provide a concrete setting in which to study the power of the demon. In the recent optical simulation by Vidrighin and coworkers [23], a two-mode optical field impinges on two photodiodes and the electric charges thence emitted are used to charge a capacitor. In the limit of ideal linear photodiodes, every photon creates an electron: therefore the voltage is directly proportional to the difference of photon number between the two modes. Initially, the two modes carry independent fields with identical photon-number distribution (chosen as thermal). In the absence of the demon, the two photodiodes produce on average the same photocurrent and the capacitor is not charged. The demon is mimicked by a weak monitoring of the two fields, realised approximately by photon subtraction and a single photon detection. Some detection patterns imply a bias

in the number of photon at a given time, which can be used to charge the capacity.

In this paper, we focus on this setup to investigate how the power of the demon varies by changing the photon-number statistics of the two fields. We mostly consider cases in which the partial state of each mode is still thermal, but the photon numbers may be correlated between the modes (not with the demon). We shall notably see that the behavior of the simulation is analog to the one expected for the thermal demon (see Fig. 1).

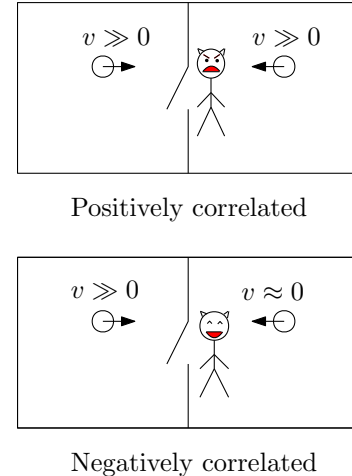


FIG. 1. For the original “thermal” demon, one doesn’t expect correlations between the fluctuations of the two halves. But were one to play that game for the sake of it, the effect of the correlations on the demon’s power is intuitive. If at the trapdoor, each time a fast molecule comes from the left there is a fast molecule coming from the right, the demon’s action will be hindered. If each time a fast molecule comes from the left there is a slow molecule coming from the right, the power of the demon is enhanced.

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II. FRAMEWORK FOR THE STUDY

A. The setup

The measurement setup is sketched in Fig. 2. Two optical modes are sent on photodiodes operated in proportional mode, and the difference in photocurrents is used to charge a capacitor. If the goal were to optimise the charging of the capacitor, one would simply leave one of the modes in the vacuum state, thus achieving maximal bias. Mimicking the Maxwell's demon thought experiment rather requires the two modes to carry the same average number of photons $\bar{n}_A = \bar{n}_B$: thus, in the absence of the demon, there will be no net charging.

An even more accurate simulation of the demon demands that the states are uncorrelated *thermal states* at the same temperature, as coming from two sources in contact with the *same* bath. Besides, like in the experiment we keep only a mono-frequency component of the blackbody radiation¹. A monomode optical field in equilibrium with a thermal bath at temperature T is in the thermal state

$$\rho^{\text{th}}(\beta) = (1 - e^{-\beta\hbar\omega}) \sum_{n=0}^{\infty} e^{-n\beta\hbar\omega} |n\rangle\langle n|, \quad (1)$$

with $\beta = 1/(k_B T)$ as usual. As well known, the average photon number in such a state is

$$\bar{n} = \frac{1}{\exp(\beta\hbar\omega) - 1} = \frac{\lambda}{1 - \lambda}, \quad (2)$$

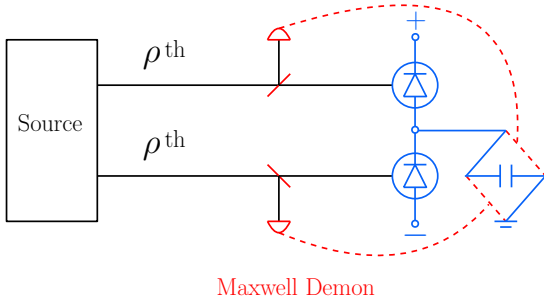


FIG. 2. (Color online) The setup under study, explained in detail in the text. It consists of the optical source and modes (black), the electronic detection circuit (blue) and the devices of the demon (red).

¹ In the experiment, the field was mono-frequency because of the way the state was prepared: not by coupling to a bath, but by a customary randomisation of an initially coherent laser beam. For the present theory, it is clear that the demon simulator under study does not distinguish between frequencies, so keeping the whole blackbody spectrum would just add unnecessary complications to the toy model. Needless to say, a demon that could sort frequencies would be more powerful.

with $\lambda = \exp(-\beta\hbar\omega)$. The state prepared in the experiment was therefore

$$\rho_{AB}^{\text{th}} = \rho_A^{\text{th}} \otimes \rho_B^{\text{th}}. \quad (3)$$

The demon is implemented as follows: a beam-splitter is put in front of each photodiode, and the reflected beams are monitored with photon counters. When the reflectivity of the beam splitter is low, this approximates a *photon subtraction* in each beam [24]. The thermal state being bunched, the fact that a photon has been subtracted *increases* the expected number of photons [25]. In other words, if one of the counters clicks and the other does not, there are on average more photons in the mode whose detector has clicked. One can then choose the polarity of the capacitor accordingly and achieve a net charging.

B. Tools for the calculation

Since both the demon's operation and the final measurement are not sensitive to coherence in the number basis, without loss of generality we can study photon statistics coming from states that are diagonal in that basis:

$$\rho_{AB} = \sum_{n_A, n_B} p(n_A, n_B) |n_A\rangle\langle n_A| \otimes |n_B\rangle\langle n_B|. \quad (4)$$

On each mode $X = A, B$, the demon's operation consists of inserting a beam-splitter

$$|n_X\rangle\langle n_X| \longrightarrow \sum_{k=0}^n \binom{n}{k} (1 - R_X)^{n-k} R_X^k \times |n-k\rangle_X\langle n-k| \otimes |k\rangle_{X'}\langle k|, \quad (5)$$

followed by photon counting on the reflected mode X' , and R_X is the reflectance of the beam splitter on mode X . The latter measurement is described by the two-outcome POVM $\{\Pi_{X'}^{(0)}, \Pi_{X'}^{(1)}\}$ where

$$\Pi_{X'}^{(0)} = \sum_{j \geq 0} (1 - \eta_X)^j |j_{X'}\rangle\langle j_{X'}|, \quad (6)$$

$$\Pi_{X'}^{(1)} = \sum_{j > 0} (1 - (1 - \eta_X)^j) |j_{X'}\rangle\langle j_{X'}| \quad (7)$$

describe the cases in which the photon counter did not, and did click, respectively, and η_X is the quantum efficiency of the counter at mode X . Both R and η are free parameters that describe the demon. We remark again that one could achieve a large bias by blocking one of the beams and letting the other go through ($R_A = 1$ and $R_B = 0$), but this could not be called a simulation of the demon. To avoid this type of bias, we shall set $R_A = R_B = R$.

For each of the four possible outcomes of the photon counting $C \equiv (c_A, c_B) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$, we compute the probability P_C that this outcome happens,

as well as $\bar{n}_{B|C} - \bar{n}_{A|C}$ on the conditional state left in the transmitted beams. If the latter average is negative, the polarity of the capacitor is switched. Thus, the figure of merit to be optimised over the parameters (R, η_A, η_B) of the demon is

$$\langle \Delta n \rangle = \sum_C (-1)^{s(C)} P_C \langle \Delta n \rangle_C, \quad (8)$$

with $\langle \Delta n \rangle_C = \bar{n}_{B|C} - \bar{n}_{A|C}$, where $s(C) = 1$ if polarity of the capacitor should be switched and $s(C) = 0$ otherwise.

III. RESULTS

For the two uncorrelated thermal states with $\bar{n}_A = \bar{n}_B = \bar{n}$ studied in Ref. [23], one finds $\langle \Delta n \rangle_{\max} = (16/27)\bar{n}$ using $s(1,0) = 1$. Indeed, as already mentioned, for a thermal state a successful photon subtraction in mode A leads to $\bar{n}_{A|C} > \bar{n}$. Before proceeding to the study of other input states, it is important to remark that this maximum value of $(16/27)\bar{n}$ is only achievable for $\bar{n} \gg 1$, which is the experimental case under study in Ref. [23]. The reason is because the optimization there was done with respect to the probabilities $p_{A/B} = \text{tr}\{\rho_{A/B}\Pi^{(1)}\}$, that there is a click in the mode A or B, rather than over R and η_X , and for small \bar{n} , not all values for $0 \leq p \leq 1$ are compatible with the constraint that $0 \leq \eta_{A/B}, R \leq 1$. Indeed one finds

$$p_X = \frac{R\eta_X\lambda}{1 - \lambda + R\eta_X\lambda}, \quad (9)$$

and therefore $0 \leq p_X \leq \lambda$. In the limit $\bar{n} \rightarrow \infty$ one has $\lambda \rightarrow 1$; but, for example if $\bar{n} = 1$, then $p_X \leq \frac{1}{2}$. In this case, the optimal value $p_A = 2/3$, required to get $\langle \Delta n \rangle_{\max} = 16/27$, cannot be reached. In Appendix A, we show $\langle \Delta n \rangle_{\max}/\bar{n}$ as \bar{n} is varied. It is about $\bar{n} \approx 50$ that the maximum value of $(16/27)\bar{n}$ can be realized.

A. Two uncorrelated thermal states at different temperatures

As a first case study, we consider a product of thermal states at different temperatures:

$$p(n_A, n_B) = (1 - \lambda_A)\lambda_A^{n_A} (1 - \lambda_B)\lambda_B^{n_B}. \quad (10)$$

where $\tilde{R}_X \equiv \bar{n}_X R \eta_X$. For an arbitrary \bar{n}_A and \bar{n}_B , the maximum of this expression with constraint that $0 \leq \eta_X, R \leq 1$ is very lengthy and only numerical values can

be obtained. Without loss of generality, we assume that the temperature of mode B is higher than that of mode A, so that $\bar{n}_B > \bar{n}_A$.

Obviously, with this state, the capacitor can be charged even without the demon. The average photon number difference created across the plates of the capacitor is simply $\langle \Delta n \rangle = \bar{n}_B - \bar{n}_A$. We want to check if the demon can still provide an advantage.

The calculation proceeds similarly as in Ref. [23], but we do not assume $\bar{n}_i \gg 1$ or $R \rightarrow 0$. The details are listed in Table I where we denoted by $p_{A/B}$ the probability that the counter on each mode clicks.

TABLE I. Uncorrelated thermal states at different temperatures. For simplicity of reading we have scaled $\langle \Delta n \rangle_C$ by a factor of $(1 - R)P_{(0,0)}$ and denoted $\delta_1 = R\bar{n}_A\bar{n}_B(\eta_A - \eta_B)$, $\delta_2 = R\bar{n}_B[(\bar{n}_B - \bar{n}_A)\eta_B + \bar{n}_A\eta_A(2 + R\bar{n}_B\eta_B)] > 0$, and $\delta_3 = R\bar{n}_A[(\bar{n}_B - \bar{n}_A)\eta_A - \bar{n}_B\eta_B(2 + R\bar{n}_A\eta_A)]$. In the table, p_X , the probability of a click at mode X, is given by Eq. (9).

C	P_C	$\langle \Delta n \rangle_C / (P_{(0,0)}(1 - R))$
(0,0)	$(1 - p_A)(1 - p_B)$	$\bar{n}_B - \bar{n}_A + \delta_1$
(0,1)	$(1 - p_A)p_B$	$2\bar{n}_B - \bar{n}_A + \delta_2$
(1,0)	$p_A(1 - p_B)$	$\bar{n}_B - 2\bar{n}_A + \delta_3$
(1,1)	$p_A p_B$	$2\bar{n}_B - 2\bar{n}_A + \delta_2 + \delta_3 - \delta_1$

Now except the case $C = (0,1)$ where $\langle \Delta n \rangle_C$ is surely positive, the other three cases could potentially be negative depending on the values of the parameters. To find the optimal strategy of the demon, we numerically maximize $\langle \Delta n \rangle$ for all eight possible combinations of $s(C)$ for the other three cases for different range of values of \bar{n}_A and \bar{n}_B . We first observed that for the situation of $\bar{n}_B \gg \bar{n}_A$, the best improvement by the demon is negligible, as it would be for the thermal demon.

Focusing then on the interesting regime where $\bar{n}_B \gtrsim \bar{n}_A$, we find that the maximum is always achieved for putting only $s(1,0) = 1$. The average photon number difference created is then

$$\langle \Delta n \rangle = (1 - R) \left[\bar{n}_B - \bar{n}_A + \frac{2\tilde{R}_A(\bar{n}_A(1 + \tilde{R}_B)(2 + \tilde{R}_A) - \bar{n}_B(1 + \tilde{R}_A))}{(1 + \tilde{R}_A)^2(1 + \tilde{R}_B)^2} \right], \quad (11)$$

be obtained. As a remark, we note that for the case $\bar{n}_A = \bar{n}_B = 1$, $\langle \Delta n \rangle_{\max} \approx 0.255 < 16/27$, achieved with $\eta_B \approx 0.427$, $\eta_A = 1$, and $R \approx 0.344$. Making connections

with Ref. [23], for the case where the average photon number in the two modes is quite large, in terms of the probabilities $p_{A/B}$ one finds the simpler expression

$$\langle \Delta n \rangle = (1 - R) \{ \bar{n}_B - \bar{n}_A + 2p_A(1 - p_B) \times [\bar{n}_A(2 - p_A) - \bar{n}_B(1 - p_B)] \}, \quad (12)$$

whose maximum value is

$$\begin{aligned} \langle \Delta n \rangle_{\max} &= (1 - R) \left(\bar{n}_B - \bar{n}_A + \frac{16 \bar{n}_A^2}{27 \bar{n}_B} \right) \\ &\approx \bar{n}_B - \bar{n}_A + \frac{16 \bar{n}_A^2}{27 \bar{n}_B}, \end{aligned} \quad (13)$$

obtained for $R = \frac{2}{\bar{n}_A}$, $\eta_A = 1$, and $\eta_B = \frac{3\bar{n}_B - 2\bar{n}_A}{4\bar{n}_B}$. In terms of the probabilities, this translates to $p_A = \frac{2}{3}$ and $p_B = 1 - \frac{2}{3} \frac{\bar{n}_A}{\bar{n}_B}$. Thus the demon's action contributes an improvement of $\frac{16 \bar{n}_A^2}{27 \bar{n}_B} \leq \frac{16}{27} \bar{n}_A$, with equality if and only if $\bar{n}_A = \bar{n}_B = \bar{n}$. In other words, as expected, the demon helps, and its help is maximal when the two thermal states have the same temperature.

B. Split thermal state

Next we consider the state obtained by sending a thermal state through a beam splitter: $\rho_{AB} = U \rho^{\text{th}}(\beta') U^\dagger$, where $U = \exp[\theta(a^\dagger b e^{i\phi} - a b^\dagger e^{-i\phi})]$ and the reflectance of the beam splitter is given by $R = \sin^2 \theta$ [26]. The photon number distribution is now given by [27]

$$\begin{aligned} p(n_A, n_B) &= \frac{1}{1 + \bar{n}_{\text{in}}} \frac{(n_A + n_B)!}{n_A! n_B!} \\ &\times \left(\frac{\bar{n}_A}{1 + \bar{n}_{\text{in}}} \right)^{n_A} \left(\frac{\bar{n}_B}{1 + \bar{n}_{\text{in}}} \right)^{n_B}, \end{aligned} \quad (14)$$

where $\bar{n}_{\text{in}} = 1/(e^{\beta' \hbar \omega} - 1)$. Using $\sum_{k=0}^{\infty} \binom{k+a}{k} x^k = (1 - x)^{-(1+a)}$, we see that the marginal states are still thermal with average number $\bar{n}_A = \bar{n}_{\text{in}} \cos^2 \theta$ and $\bar{n}_B = \bar{n}_{\text{in}} \sin^2 \theta$, but now there are correlations (in particular, it holds $\langle n_A n_B \rangle = 2\bar{n}_A \bar{n}_B$).

The result of the calculation is presented in Table II. The expressions are unpleasant but one feature is clear: all the $\langle \Delta n \rangle_C$ are proportional to $\bar{n}_B - \bar{n}_A$ with a positive factor. In other words, because of the correlations created by the beam-splitter, the sign of the difference of photon numbers is not modified for any value of C . The demon cannot help when such correlations are present and one checks that $\langle \Delta n \rangle = (1 - R)(\bar{n}_B - \bar{n}_A)$.

C. Number-correlated state (two-mode squeezed state)

Next we consider the number correlated state

$$p(n_A, n_B) = \frac{1}{1 + \bar{n}} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^{n_A} \delta_{n_A, n_B}. \quad (15)$$

TABLE II. Split thermal state. We denote $\tilde{R}_X \equiv \bar{n}_X R \eta_X$. To facilitate reading, we have scaled the expressions by $P_{(0,0)} = 1/(1 + \tilde{R}_A + \tilde{R}_B)$ and denoted $K \equiv 1 + 1/P_{(0,0)} = 2 + \tilde{R}_A + \tilde{R}_B$. The expression for $\langle \Delta n \rangle_C$ is also scaled by $1 - R$. In the table, $K' = 2(K - 1)[3 + \tilde{R}_A \tilde{R}_B + (\tilde{R}_A + \tilde{R}_B)(\tilde{R}_A + \tilde{R}_B + 3)] + \tilde{R}_A \tilde{R}_B (\tilde{R}_A + \tilde{R}_B)^2$.

C	$P_C/P_{(0,0)}$	$\langle \Delta n \rangle_C / (P_{(0,0)}(1 - R))$
(0,0)	1	$(\bar{n}_B - \bar{n}_A)$
(0,1)	$\frac{\tilde{R}_B}{(1 + \tilde{R}_A)}$	$\frac{(\bar{n}_B - \bar{n}_A)(K + \tilde{R}_A)}{(1 + \tilde{R}_A)}$
(1,0)	$\frac{\tilde{R}_A}{(1 + \tilde{R}_B)}$	$\frac{(\bar{n}_B - \bar{n}_A)(K + \tilde{R}_B)}{(1 + \tilde{R}_B)}$
(1,1)	$\frac{\tilde{R}_A \tilde{R}_B K}{(1 + \tilde{R}_A)(1 + \tilde{R}_B)}$	$\frac{(\bar{n}_B - \bar{n}_A) K'}{(1 + \tilde{R}_A)(1 + \tilde{R}_B) K}$

This distribution is that of the two-mode squeezed state

$$|\psi\rangle_{AB} = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |n_A, n_B\rangle, \quad (16)$$

where r is the squeezing parameter and $\bar{n} = \sinh^2 r$. For this state, the calculations are heavy and the final optimisation must be done numerically; so we give them in Appendix B. The overall result is that the demon helps but very little, with

$$\langle \Delta n \rangle_{\max} \approx 0.7, \text{ for } \bar{n} \rightarrow \infty. \quad (17)$$

It is instructive to give a qualitative understanding of this result. Recall that the demon basically performs photon subtraction. Upon detecting a photon in (say) the reflection of mode A, as before the demon knows that probably there are several photons left in that mode, because of the marginal thermal statistics. But now it also knows that in the other mode there are many photons too: to be precise, there is *one more photon*. So, now the demon has to set $s(0, 1) = 1$, instead of $s(1, 0) = 1$ as for the uncorrelated thermal states; besides, the difference in photon numbers will never be larger than 1 on average. Finally, as comparison to the other states, we note that for $\bar{n} = 1$, $\langle \Delta n \rangle_{\max} \approx 0.272$, which is achieved with $R \approx 0.373$, $\eta_A = 1$, and $\eta_B \approx 0.415$.

D. Number-anticorrelated states

The previous result immediately evokes its counterpart: it is clear that the demon will be very efficient if the modes are *anti-correlated* in numbers, for instance in a mixture of $|n_A, 0_B\rangle$ and $|0_A, n_B\rangle$. We are therefore going to study states with statistics

$$p(0, 0) = q_0, \quad p(n, 0) = p(0, n) = q_n/2, \text{ for } n > 0, \quad (18)$$

and all the other $p(n_A, n_B) = 0$. The normalisation condition is $\sum_n q_n = 1$. For all these states, $P_C = 0$ for $C = (1, 1)$; and, as soon as one counter clicks, the demon knows that that mode contains all the remaining photons.

Let us first consider the simple case $q_k = \frac{1}{2}\delta_{k,m}$ with $\bar{n}_A = \bar{n}_B = m/2$. When one of the counters clicks [$C = (0, 1)$ or $(1, 0)$], the value of $P_C\langle\Delta n\rangle_C$ in the corresponding mode is

$$\sum_{k=0}^m R^k (1-R)^{m-k} (1 - (1-\eta)^k) (m-k) = m(1-R) [1 - (1-R\eta)^{m-1}]. \quad (19)$$

When neither counter clicks, if $\eta_B \leq \eta_A$ one can still have an estimate $\langle\Delta n\rangle_{(0,0)} \geq 0$: indeed, for $C = (0, 0)$,

$$P_C\langle\Delta n\rangle_C = \frac{m(1-R)}{2} \times [(1-R\eta_B)^{m-1} - (1-R\eta_A)^{m-1}]. \quad (20)$$

All in all, by setting $s(1, 0) = 1$, the demon achieves

$$\langle\Delta n\rangle = m(1-R)[1 - (1-R\eta_A)^{m-1}], \quad (21)$$

with $\eta_A \geq \eta_B$. For $m = 0, 1$, this yields 0 as it should. Maximising over the demon's parameters, one finds

$$\langle\Delta n\rangle_{\max} = (m-1) m^{\frac{1}{1-m}} \approx m-1 - \ln m, \text{ for } m \gg 1, \quad (22)$$

achieved for $\eta_A = 1$ and $R = 1 - m^{\frac{1}{1-m}}$. Recalling that $m = 2\bar{n}$, on this state the demon comes close to the absolute maximal performance, as expected.

After this simple example, let us study number-anticorrelated states whose marginals are thermal. From Eq. (18), we have $p(n_{A/B} = n) = q_n/2$ for $n > 0$, and therefore we want to impose $q_n = \frac{2}{1+\bar{n}} \left(\frac{\bar{n}}{1+\bar{n}}\right)^n$. This implies $\sum_{n>0} q_n = 2(1 - 1/(1+\bar{n}))$. Checking that this sum is smaller than one constrains the mean photon number in each mode to satisfy $\bar{n} \leq 1$.

Let us finish the calculation for the case $\bar{n} = 1$, corresponding to $q_0 = 0$ and $q_{n>0} = 2^{-n}$. With the results obtained above, it is straightforward to work out the mean photon number difference

$$\langle\Delta n\rangle = \frac{2(1-R)R\eta_A(2+R\eta_A)}{(1+R\eta_A)^2}. \quad (23)$$

The function is maximised for $\eta_A = 1$ and for R solution of the equation $R^3 + 3R^2 + 4R - 2 = 0$, i.e. $R \approx 0.379$. The corresponding value for the maximum photon number difference is then the solution of the equation $4x^3 - 49x^2 + 272x - 144 = 0$, that is

$$\langle\Delta n\rangle_{\max} \approx 0.589. \quad (24)$$

E. Quick overview

Table III summarizes some of the results, namely the maximal contribution of the demon to the states that we studied, in the case $\bar{n}_A = \bar{n}_B$. The number-anticorrelated

TABLE III. Summary: the contribution of the demon for the states studied in this section, with $\bar{n}_A = \bar{n}_B = \bar{n}$. The first four states have thermal marginals.

State	$\langle\Delta n\rangle_{\max}$ ($\bar{n} = 1$)	$\langle\Delta n\rangle_{\max}$ (Large \bar{n})
Uncorrelated	≈ 0.255	$(16/27)\bar{n}$ [23]
Split thermal state	0	0
Number-correlated	≈ 0.272	≈ 0.7
Number-anticorrelated	≈ 0.589	–
Number-anticorrelated (m)	0.5	$\approx 2\bar{n}$

state give the demon the highest power, the number correlated ones hinder its action. Thus, the optical demon behaves in good analogy with the thermal demon (Fig. 1) even in this respect. Among the states with thermal marginals, in the limit of large \bar{n} , the original uncorrelated state seems to be the best option, insofar as anti-correlated states are not available.

IV. REMARKS IN THE SPIRIT OF A RESOURCE THEORY

The *resource theory of thermodynamics* has been built on very general premises, inspired by entanglement theory [28–32]. A state is called *passive* if its energy cannot be decreased by applying the most general free operations. The idea is that a decrease of energy in the state may be exploited to create work. A non-passive state is a resource. In this general framework, it was proved that the only completely passive state (i.e. a state that remains passive no matter how many copies of it are made available) is the thermal state.

These general results cannot be exported as such to our study, but an analysis in terms of resources may nevertheless be attempted. After all, the choice of thermal state in the original paper was motivated by its passivity; and the demon is definitely a resource, so it may be interesting to compare it to other resources.

Let us first assume that the goal is not to extract work through the most general possible operation, but just to charge the capacitor in the scheme under consideration (in particular, we stick to two optical modes). Further, let us assume that the only free operation is the coupling of an optical mode with a thermal bath at temperature T with mean photon number \bar{n}_T , resulting in the thermalisation of the partial state [33].

With these rules, the passive states are not only the thermal states, but all those such that $\bar{n}_A = \bar{n}_B = \bar{n}_T$. Indeed, if a state has $\bar{n}_A \neq \bar{n}_B$, a net charge can be created. And if a state has $\bar{n}_A = \bar{n}_B \neq \bar{n}_T$, one can thermalise one of the modes for free, thus creating $\bar{n}_A \neq \bar{n}_B$.

With respect to this classification, the uncorrelated thermal state of the original study is still passive, as it should. The split thermal state is obviously not passive, because $\bar{n}_A = \bar{n}_B = \bar{n}_T/2$, so one can thermalise

mode B to reach $\bar{n}_B = 2\bar{n}_A$. Interestingly, recall that for this state the demon as implemented doesn't help at all. Thus, as a resource, *the demon is incommensurable with state preparation*. Turning to the other states (number-correlated and -anticorrelated), their definition does not involve any thermal bath; so the choice of a T for the bath is uncorrelated from the parameters of the state. Unsurprisingly, we find that all states in these families are resources, insofar as one does not choose T to match exactly $\bar{n}_A = \bar{n}_B = \bar{n}_T$.

It goes without saying that these few lines lack both the depth and the sophistication of a proper resource theory. We find it nonetheless interesting to notice that the main features of such a theory are neatly recovered.

V. CONCLUSION

We have pushed the study of the optical simulation of the Maxwell demon demonstrated in Ref. [23]. The obvious fact is that the information that the demon collects must be useful. In the original study, it was useful because it could signal a change in statistics in one and only one of the modes. We have seen that the usefulness decreases significantly if the numbers are positively correlated across the modes, and increases if they are negatively correlated. This is the same behavior a thermal demon would exhibit in the presence of similar correlations

in the speed of the atoms between the two reservoirs.

On the one hand, there are some quantum elements in the demon under study: notably, it uses photon-counting, which cannot be described in a classical theory of light. Also, one may say that number anti-correlated states of optical fields ($g^{(2)} < 1$) are necessarily quantum, and this demon's power is enhanced by them. On the other hand, though, all the statistics we studied could be simulated with classical systems: quantum correlations do not play any role for this demon. A practical implementation of a properly quantum demon, that would allow studying behaviors like those predicted in some information-theoretical papers [9–11], is still lacking.

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Appendix A: Two uncorrelated thermal states with the same mean photon number

For small values of \bar{n} , one cannot achieve the maximum value of $(16/27)\bar{n}$ with the measurement of the demon. Fig. 3 below shows the optimal value that could be reached with the constraints $0 \leq R, \eta_X \leq 1$. One observes that

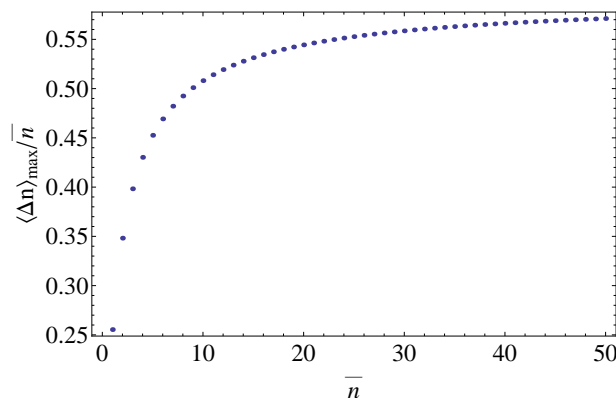


FIG. 3. Graph of maximum photon number difference that can be created by the Maxwell's demon when the average photon number in the two independent thermal state is varied.

the maximum value can be reached when $\bar{n} \approx 50$.

Appendix B: Details for the number-correlated state

In this Appendix, we give details on the calculations for the number-correlated state presented in Section III C. With a slightly different notation from the main text here that $\tilde{R}_X = \eta_X R$, the probabilities for the counting patterns of the demon are:

$$P_{(0,0)} = \frac{1}{1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)}, \quad (\text{B.1})$$

$$P_{(0,1)} = \frac{\bar{n} \tilde{R}_A (1 - \tilde{R}_B)}{(1 + \bar{n} \tilde{R}_A)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]}, \quad (\text{B.2})$$

$$P_{(1,0)} = \frac{\bar{n} \tilde{R}_B (1 - \tilde{R}_A)}{(1 + \bar{n} \tilde{R}_B)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]}, \quad (\text{B.3})$$

$$P_{(1,1)} = \frac{\bar{n} \tilde{R}_A \tilde{R}_B [1 + 2\bar{n} + \bar{n}^2(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]}{(1 + \bar{n} \tilde{R}_A)(1 + \bar{n} \tilde{R}_B)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]}. \quad (\text{B.4})$$

The mean photon number difference conditioned on each case is given by

$$\langle \Delta n \rangle_{(0,0)} = \frac{\bar{n}(1 - R)(\tilde{R}_A - \tilde{R}_B)}{1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)}, \quad (\text{B.5})$$

$$\langle \Delta n \rangle_{(0,1)} = -\frac{(1 - R) \left(1 + \bar{n} \tilde{R}_A \left[4 - 2\tilde{R}_A + \bar{n} \left(\tilde{R}_A(3 - \tilde{R}_A) + \tilde{R}_B(1 - \tilde{R}_A)^2 \right) \right] \right)}{(1 - \tilde{R}_A)(1 + \bar{n} \tilde{R}_A)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]} < 0, \quad (\text{B.6})$$

$$\langle \Delta n \rangle_{(1,0)} = \frac{(1 - R) \left(1 + \bar{n} \tilde{R}_B \left[4 - 2\tilde{R}_B + \bar{n} \left(\tilde{R}_B(3 - \tilde{R}_B) + \tilde{R}_A(1 - \tilde{R}_B)^2 \right) \right] \right)}{(1 - \tilde{R}_B)(1 + \bar{n} \tilde{R}_B)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]} > 0, \quad (\text{B.7})$$

and

$$\langle \Delta n \rangle_{(1,1)} = \frac{\bar{n}(1 - R)(\tilde{R}_A - \tilde{R}_B)(2 + \bar{n}f_1 + \bar{n}^2f_2 + \bar{n}^3f_3)}{(1 + \bar{n} \tilde{R}_A)(1 + \bar{n} \tilde{R}_B)[1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)][1 + 2\bar{n} + \bar{n}^2(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B)]}, \quad (\text{B.8})$$

with

$$f_1 = 3 + 2\tilde{R}_A + 2\tilde{R}_B - \tilde{R}_A \tilde{R}_B, \quad (\text{B.9})$$

$$f_2 = 2(2\tilde{R}_A + 2\tilde{R}_B - \tilde{R}_A \tilde{R}_B), \quad (\text{B.10})$$

$$f_3 = \tilde{R}_B^2(1 - \tilde{R}_A)^2 + \tilde{R}_A \tilde{R}_B(3 - 2\tilde{R}_A) + \tilde{R}_A^2. \quad (\text{B.11})$$

That the expression for $\langle \Delta n \rangle_{(0,1)}$ is always negative and $\langle \Delta n \rangle_{(1,0)}$ always positive reflects that the demon knows, upon detection of a photon in one mode, there is probably one more photon in the other mode. Hence, one should set $s(0, 1) = 1$, and $s(1, 0) = 0$. Due to the symmetry of the two modes, one can choose $\eta_A > \eta_B$ without loss of generality. It follows then both $\langle \Delta n \rangle_{(0,0)}$ and $\langle \Delta n \rangle_{(1,1)}$ are positive and $s(0, 0) = s(1, 1) = 0$. With this optimal strategy, the expression of the mean photon number difference created by the demon with feed-forward is

$$\langle \Delta n \rangle = \frac{2\bar{n}(1 - R)\tilde{R}_A[1 + 2\bar{n}\tilde{R}_B(2 - \tilde{R}_B) + \bar{n}^2(\tilde{R}_B^2(3 - 2\tilde{R}_B) + \tilde{R}_A \tilde{R}_B(1 - \tilde{R}_B)^2)]}{(1 + \bar{n} \tilde{R}_B)^2(1 + \bar{n}(\tilde{R}_A + \tilde{R}_B - \tilde{R}_A \tilde{R}_B))^2}. \quad (\text{B.12})$$

However, this expression is complicated so one cannot obtain an analytical expression for $\langle \Delta n \rangle_{\max}$ and the corresponding values for R and the η 's. Instead, we numerically maximize this expression subject to the necessary constraints that the reflectivity and detector efficiencies are between 0 and 1, and Fig. 4 shows the result.

Finally, as a comparison with the other states, we note that when $\bar{n} = 1$, we have $\langle \Delta n \rangle_{\max} \approx 0.272$, which is slightly larger than the value of 0.255 obtained for uncorrelated thermal states. This is achieved with $R \approx 0.373$, $\eta_A = 1$ and $\eta_B \approx 0.415$.

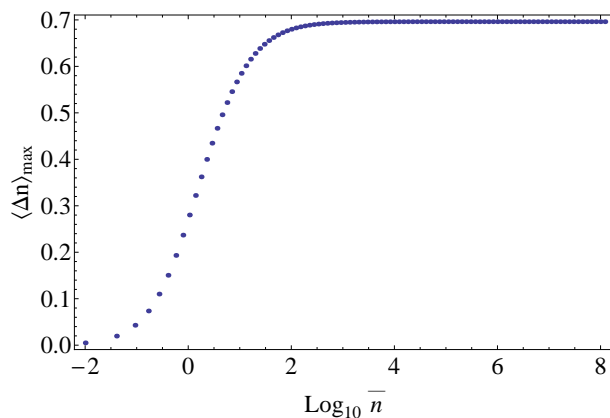


FIG. 4. Graph of maximum photon number difference that can be created by the Maxwell's demon when the average photon number in the number-correlated state is varied.

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